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Modern Signal Processing Technique for Optimal Signal to Noise Ratios

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Phase sensitive detection provides the theoretically optimum technique for the recovery of signal intensity information from noise. Application of this technique to signal processing problems encountered in typical experiments in physics, chemistry, astronomy and biology is discussed.

It is probably a safe statement to make that the majority of present day research in the physical sciences involves the measurement of small effect phenomena where noise sets the limit to attainable precision or detectability. In noise one includes all disturbing elements over which the experimentalist has no control, such as effects produced by the fundamental thermal fluctuations of all matter not at absolute zero, or by statistical fluctuations due to the quantized nature of light, electrical currents, etc. Also in noise one groups such extraneous disturbances as building vibrations, variations in room temperature and stray electrical signal pickup; disturbances which in principle can be reduced to an arbitrarily small value but which in practice are difficult to remove entirely from the picture.

The experimentalist who has pursued a phenomenon of interest down to the noise threshold is continually alert for methods that may improve the signal-to-noise ratio (S/N) of his measurements. Such a method is the lock-in technique, which in many cases can improve experimental S/N by 40 db or more.

Since World War II, signal processing techniques that approach attainment of the theoretically optimum signal-to-noise ratio have been developed. Some of these techniques were applied in those spectacular experiments in which radar signals reflected from Venus and the moon were detected. While these experiments were very complex, involving the use of high speed digital computers to recover the desired information from noise, the underlying principles are relatively simple and can easily be applied to more mundane research measurements. A few experimentalists have in fact been using these techniques for some time, but there are still many researchers who do not yet realize the power they provide in improving S/N.

A factor that has worked against the general application of the lock-in technique has been the absence of a commercially-available "black box" that would make it unnecessary for the individual experimentalist to design his own lock-in detection system. The recent introduction of commercially available general purpose lock-in amplifiers should facilitate the introduction of this powerful technique into many research and development laboratories, and should be particularly helpful to those individuals who are skilled in a particular scientific discipline but who are not especially familiar with electronic information processing.

Let us now consider the principles of small-effect measurement and see how S/N may be optimized. We shall be concerned here with those cases in which the quantities to be detected or measured can be made to appear as electrical signals. Fortunately most physical quantities can in fact be converted to electrical signals by suitable transducers, such as photoelectric cells, strain gages, microphones, bolometers, etc. A hypothetical experiment in which a small effect physical quantity Q(t), is to be measured as it changes with time is shown in Figure 1 in block diagram form. For example it may be a very weak source of light from a variable star or from phosphorescent bacteria, but the technique is general and can be made to apply to almost any measurement. The S/N one obtains in a given system is determined by the nature of the signal and noise sources and by the properties of the various components of the system, in particular the bandwidths employed.

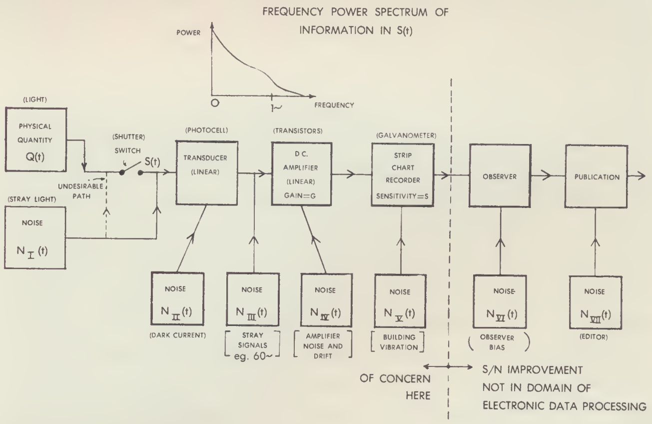

FIG. 1 Hypothetical experiment to measure and record the small-effect physical quantity Q(t) as it changes with time. Various source of noise are shown. Also shown is the frequency power spectrum of S(t).

In the traditional way of carrying out such a measurement the system might consist of a linear transducer that converts the quantity being measured into a D.C. electrical signal which is then amplified by an amplifier with D.C. response and recorded on a strip chart recorder. An important part of the system is the switch S which is opened periodically to observe system zero drift. It is very necessary to insure that the switch interrupts *only* the quantity being measured and not some of the noise sources. A typical recording from such a set-up might look like Figure 2. Here, an attempt has been made to draw in smooth curves to represent "true" zero and signal. The S/N in this example is somewhat greater than unity at the start of the recording but becomes less than one toward the end.

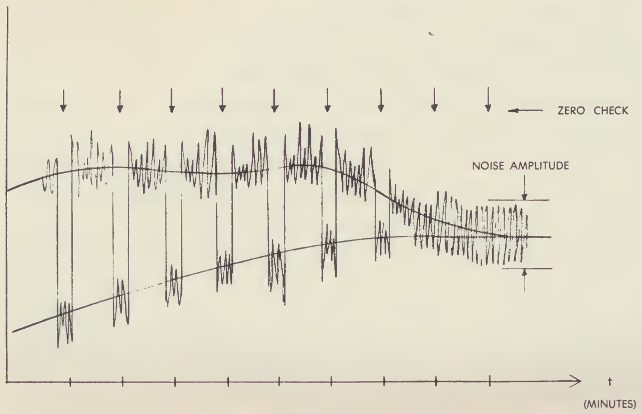

FIG. 2. Hypothetical record from experiment of Fig. 1.

Now what are the S/N considerations? The unknown is a slowly varying function of time, Q(t), which is periodically interrupted (made zero). The transducer converts this into a corresponding electrical signal. We may speak of the frequency power spectra of the quantities Q(t), or E(t) (with their periodic interruptions). These are found simply by determining the Fourier components of the input signals. The signal power in the example under study is to be found in a small band around zero frequency. Now the bandwidth of the amplifier in this system should be chosen to match this signal spectrum, i.e., it should have a corresponding bandwidth about zero frequency. The resulting S/N will then be determined by the ratio of signal power to noise power within this bandwidth. All the sources of noise in front of, and in, the amplifier must be considered. Of course the response of the recorder to sources of noise after amplification is independent of the amplifier characteristics.

Unfortunately for S/N considerations in this example, the choice of a band about zero frequency as the location for the signal power is a singularly bad one, for conventional power amplifying devices have noise power spectra that vary as the reciprocal of frequency near zero frequency (see Figure 3) due to the so-called flicker effect. Attempting to improve S/N by reducing bandwidth here is futile.

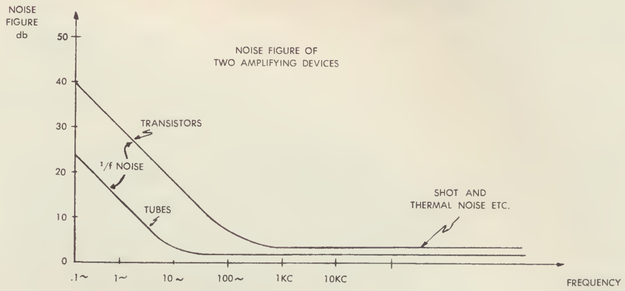

FIG. 3 Representative noise figures of two amplifying devices, the vacuum tube and the transistor, showing the characteristic l/f increase at low frequencies.

To avoid the region of l/f noise the signal information should be made to appear at some frequency, f_o, above 100 cps for tube amplifiers and above 1 kc for transistor systems. It may be desirable to go to even higher values if the signal source Q(t) has high frequency components associated with it (i.e. is rapidly varying). It may be necessary to go to lower frequencies if other necessary components in the system do not have a high enough frequency response. The signal power may be made to appear at any arbitrary frequency f_o by opening and closing the switch S (in Figure 1) at that frequency. In reality one would use a mechanical chopper, sector disc light chopper, or arrange in some way to have the desired quantity Q(t) modulated at f_o by the application of an electric current, magnetic field, or mechanical stress of that frequency (or at one half f_o for quadratic effects). The rest of the system must now be made compatible with the fact that the signal power is now contained in a band about f_o and its multiples, the bandwidth being determined by the high frequency cut-off of the Fourier spectrum of Q(t). (Actually most of the signal power is to be found in the band about the fundamental, f_o, and harmonics can generally be neglected.) First, the transducer employed to convert Q(t) into E(t) must be capable of responding to f_o. Second, the amplifier of gain G employed to boost signal power need only respond to the narrow band about f_o containing the signal power. Finally, we must convert the amplified signal power G•E(t) into a D.C. current of magnitude proportional to the amplitude of the signal, and record this detected signal on a strip chart recorder. The S/N achievable with a system of this type is dependent

only on the bandwidth of the amplifier-detector employed and on the nature of the noise sources.

A hypothetical experiment in which this philosophy of information handling is used is shown in Figure 4. Here the problem is to measure the weak phosphorescence induced in a sample by exciting radiation over a wide range of wave-lengths. We shall now consider this experiment in detail from the standpoint of optimum S/N.

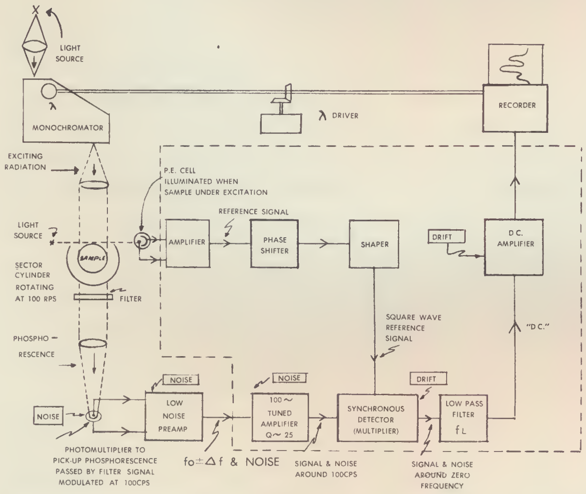

FIG. 4 Hypothetical experiment to determine the exciting radiation wave length dependence of the weak phosphorescence of a sample. Here lock-in signal processing techniques are used. Components enclosed in dotted area make up a Lock-In Amplifier.

1) A trivial observation is that the signal should be made as large as possible by using as high intensity an exciting source as possible and that the collection efficiency (f-number) of the optics should be maximized.

2) Stray light—particularly stray light that may be modulated at 100 cps by the rotating shutter system—should be strenuously avoided. Thus it is obvious that the modulation should be performed in such a way that ideally only the light due to the phosphoresence of the sample is modulated. Steady light on the detector (photomultiplier), while not contributing to signal, will increase the noise because the noise contribution due to all D.C. components of the photomultiplier current increases as the square root of that current (shot effect).

3) A photomultiplier should be used that has good quantum efficiency at the wave length under study (again large signal) and a good noise figure (low dark current, leakage, etc). As much gain as possible should be taken in the photomultiplier because, in general, the amplification obtained in the secondary emission process is about optimum from the noise standpoint. If a pre-amp is needed to follow the photomultiplier, the signal level from the latter should be made many times the inherent noise of the preamplifier, if at all possible.

The information at this point is contained in the 100 cps electrical signal from the photomultiplier and preamplifier. Actually the information is in a narrow band about the frequency $f_o=100$ cps if the quantity being measured shows any variation with time (as it will in our hypothetical system because of the changing excitation wave length). The *noise* in the channel will consist of 60 and 120 cycle pick-up from the power lines, broad band noise from the shot effect in the detector and preamplifier, flicker-effect noise from the amplifier, microphonic pick-up, etc. There remains the problem of picking the 100 cps signal out of all this noise and measuring and recording its magnitude. Continuing our consideration of S/N:

4) The 100 cps signal must now be brought to a level sufficiently high to drive the detector that will convert the information to D.C. This level should be such that the inherent drift in the detector and following D.C. amplifier are of negligible magnitude compared to the D.C. signal. In general this amplification is best done in a tuned amplifier of moderate Q. While not contributing to the overall system S/N, the tuned amplifier helps prevent overloading of the detector by random noise (through reducing the bandwidth of the noise at the detector input) or by spurious signals (e.g. 60 and 120 cps pick up).

5) The detection of the 100 cps signal is the most critical part of the system from the standpoint of S/N. The detector should have as narrow a bandwidth as possible consistent with the fact that bandwidth sets limits on the rapidity with which the amplitude of the initial unknown signal may be allowed to change and also sets limits on the observation time required to make the measurement. In going to a very narrow bandwidth detector, provision must be made to prevent the center frequency of the detector from drifting off f_o. Fortunately the lock-in detector, or synchronous detector as it is sometimes called, is available for this application. The bandwidth can be made as narrow as desired in a simple way (by increasing an RC time constant) and the center frequency is "locked-in" to the carrier frequency, f_o, avoiding drift problems. The synchronous detector is essentially a mixer that multiplies the unknown signal, $f_o \pm \Delta f$, by the "reference signal", a pure square wave signal at exactly f_o which is phase related in a definite way to the unknown signal. This mixing results in the unknown information appearing in a band of frequencies $\pm\Delta f$ about zero frequency (D.C.). (The upper side band at 2 f_o is of no interest here and is stopped by the low pass filter which follows the detector). The reference signal is obtained in a way that makes it unambiguous in frequency and phase with respect to the signal modulation wave-form. The bandwidth of the detector may be made arbitrarily narrow by passing the "zero

frequency" output through a RC low pass filter. The effective bandwidth of the detector will then be $f=1/2RC$ cycles.

5) The D.C. output of the detector must now be brought up to a level that will drive stripchart recorders. Any drift in the output of the detector or D.C. amplifier will be another source of "noise" in the experiment.

Figure 5 shows in diagrammatic form the handling of the information in our hypothetical experiment. It shows the original information at D.C., the moving of the signal to $f_0=100$ cps, amplification, and finally demodulation and filtering.

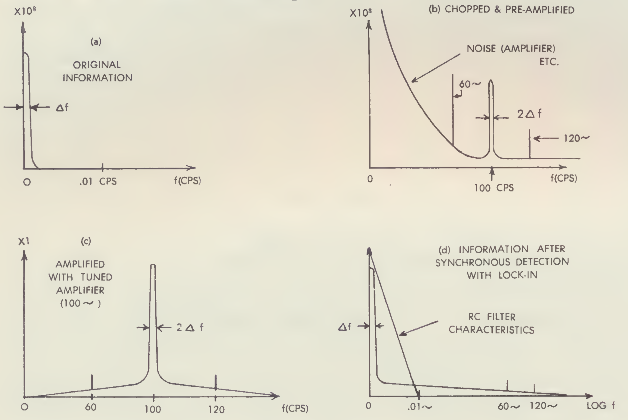

FIG. 5 "Spectra" showing how information signal is handled in hypothetical experiment of Fig. 4. By moving original information at D.C. to 100 cps. it is possible to amplify with the introduction of a minimum amount of noise.

It should be noted that, apart from D.C. drift in the synchronous detector and following amplifier, the system should be free from drift. Because of the interrupting action at the light chopper there is effectively a "zero" taken 100 times a second. However, any pick-up of 100 cps signals in the system will result in an "offset" of the output. This may be checked for by removing the sample. In practice the drift in the detector and D.C. amplifier can easily be made negligibly small.

Commercial lock-in amplifiers[1] which have the essential components of a complete signal processing system have recently become available. These employ the techniques that have been discussed above and allow the experimentalist to achieve essentially optimum S/N in his measurements. Accompanying Figures 6 and 7 show the block diagram and front panel of one such unit. (The Model JB-5 Lock-In Amplifier manufactured by Princeton Applied Research Corporation, Princeton, New Jersey) These units are very flexible in their

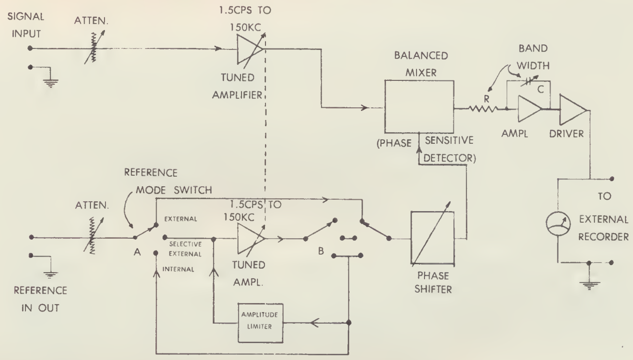

FIG. 6 Block diagram of complete signal processing unit lock-in amplifier Model JB-5.

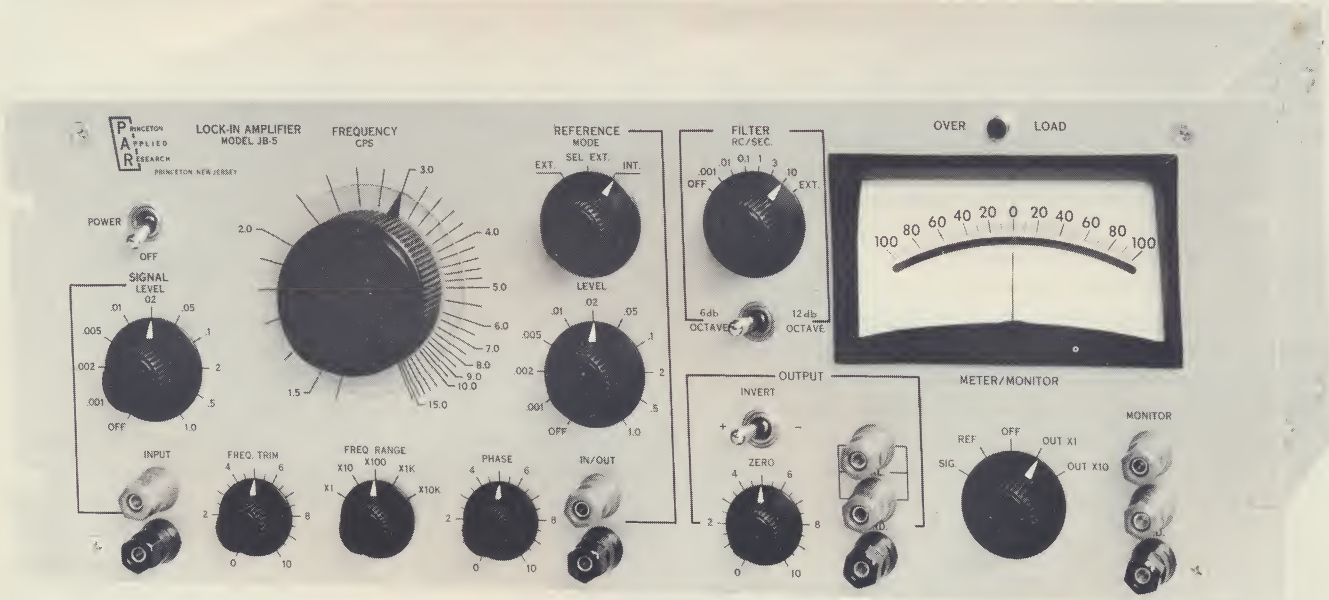

FIG. 7 Model JB-5 lock-in amplifier.

[1] R.D. Moore, Electronics, June 8, 1962

application and, in general, only require being connected to a suitable transducer and to a recorder to achieve a complete signal processing system. The unit shown has a built-in oscillator and can thus produce a reference signal which is used internally for demodulation and is available externally for use as the source of the modulation for the unknown signal.

Figure 8 shows how the commercial lock-in might be used in an experiment in biology. The experiment is purely imaginary but is fairly representative of what might be done. It illustrates the method in which the signal to be measured is generated in response to a reference voltage that is produced in the lock-in unit itself.

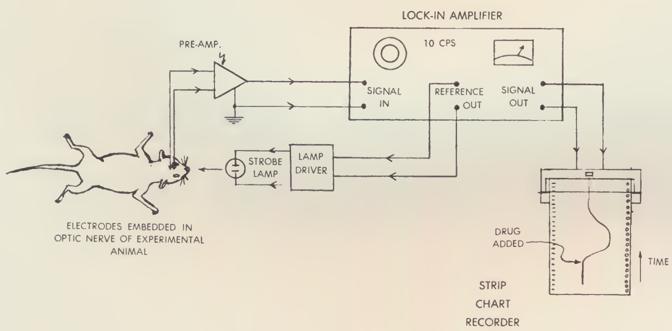

Fig. 8 Hypothetical experiment to test the effect of drug on the strength of the optical signal induced in experimental animal by 10 cps flashing light.

The strobe light is made to flash at a 10 cps rate in synchronism with the 10 cps signal from the lock-in unit. This flashing light induces 10 cycle signals in the optic nerve of the experimental animal which is picked-up by embedded electrodes. The faint electrical signals are preamplified, fed to the lock-in unit where they are further amplified by an amplifier tuned to 10 cps, detected in a synchronous detector, filtered, and recorded. The action of drugs on this signal may be studied by observing the amplitude of the recorded signal.

A classic example of the use of the lock-in signal processing technique is that of the Dicke Radiometer.[2] With this radiometer Dicke was able to measure the radiation temperature of the sky, sun, moon, and terrestrial objects to high precision. A block diagram of such a radiometer is shown in Figure 9.

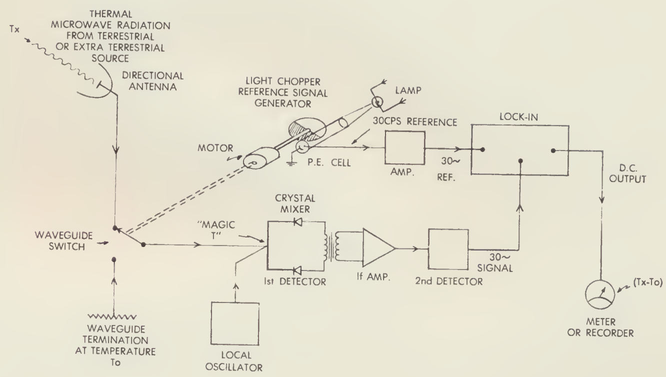

Fig. 9 Radiometer of the Dicke type.

In the foregoing, the lock-in amplifier technique has been discussed in terms of its operating principles, and a few examples of applications have been given. The applications of this technique are, in general, limited only by the ingenuity of the experimenter. In many cases a commercially available lock-in amplifier can be added to an existing piece of apparatus with good and sometimes even spectacular results. This will be particularly true if the existing apparatus makes use of D.C. detection methods. However in order to make the best possible use of this technique it is generally necessary to design the whole experiment around it. Such questions as the best method of modulation (what to modulate and how), the type of transducer to be used, the optimum operating frequency and many more must be considered with great care. In order to answer these questions correctly the experimenter must be equipped with an understanding of the lock-in amplifier technique as well as of the nature and behavior of the various factors that introduce noise, and that finally form the fundamental limitations of his experiment. The experimenter who makes the effort to acquire a sound understanding of these factors will be rewarded by a broadening of his research horizons and by a clearer insight into the scope of his experimental techniques.

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[2] R.H. Dicke, Rev. Sci. Inst. *17*, 268 (1947)

NOTES

Lock-in Amplifiers for Signals Buried in Noise

By

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Phase sensitive detector followed by low pass filter is heart of lock-in amplifier. Signals 40 db below the input noise level of a microwave receiver can be recovered with lock-in technique; oscillators can be checked to high precision quickly

SIGNAL INTENSITY measurements can be made where noise would otherwise rule them out by using a lock-in amplifier. Applications to date include radio astronomy, nuclear magnetic resonance and solid state investigations.

A lock-in amplifier is essentially a narrow band detection system in which a signal is beat with a reference signal of the same frequency, giving a d-c output. The heart of the lock-in amplifier is a phase sensitive or synchronous detector, essentially a balanced mixer. The upper side band derived from the mixer is of no interest and is stopped by a low pass filter. The lower side band (d-c) is passed by the low pass filter, the band width of which determines the band width of the amplifier. Sometimes the lock-in amplifier is operated with the reference frequency differing from the signal frequency by as little as 0.1 cps with difference frequency output recorded directly on a strip chart recorder.

While the detector elements that function as the mixer in a synchronous detector are basically nonlinear, the mixer itself functions as a linear device; for purposes of transient response or noise rejection the narrow bandpass achieved in the low-pass filter is completely equivalent to a corresponding bandwidth before mixing. Thus noise rejection with this technique is excellent.

Assume a balanced mixer is followed by a filter with a pass band from 0 to 10 cps, and that the reference frequency is 1,000 cps. If the signal frequency is between

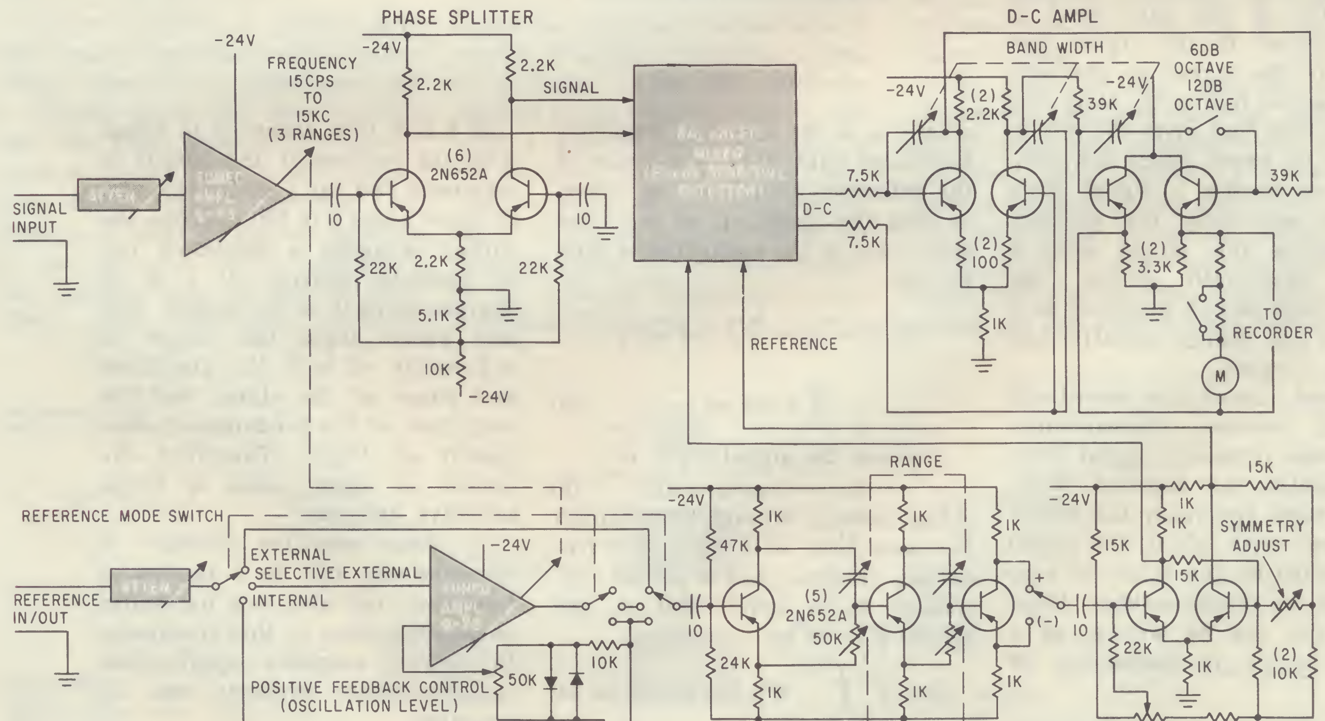


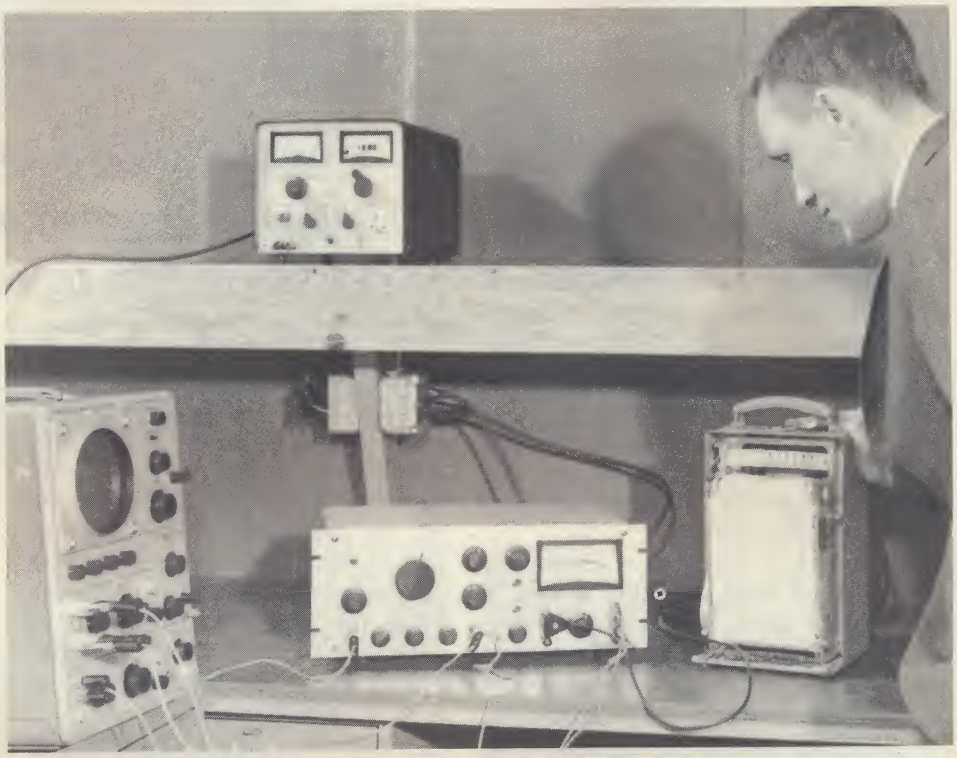
FIG. 1—Heart of lock-in amplifier is phase-sensitive detector or balanced mixer. The reference signal channel can accept an external reference or be used as a local oscillator to provide both reference and signal frequencies

990 and 1010 cps, an output signal is obtained; signals outside this band are attenuated by the low pass filter. Regardless of how the center frequency is shifted (in this example it is set at 1 Kc), the output is always the narrow band of frequencies passed by the low pass filter.

The circuit has several advantages over a conventional tuned amplifier followed by a single square law detector. First, it is easily tunable since the reference signal determines the center frequency of the pass band. Second, no matter how narrow the bandwidth of the detection system, the center of the pass band is always locked to the signal frequency if the signal is available for use as the reference. It is this characteristic that gives the lock-in amplifier its name. Since the problem of recovering a signal from noise is essentially the problem of detecting the signal with a narrow bandwidth device, the narrow band lock-in amplifier is a powerful and highly versatile tool for this purpose.

Balanced mixers are essentially nonlinear devices. Consequently, even if the reference signal $R(t)$ is sinusoidal, the function $R'(t)$ (at the same frequency but different phase) with which the signal is multiplied by the mixer in general will be nonsinusoidal. However, $R'(t)$ can be written as a Fourier series in harmonics of $R(t)$

$$R'(t) = A_0 + \sum_{1}^{\infty} A_n \cos n\omega t + \sum_{1}^{\infty} B_n \sin n\omega t \qquad (1)$$



Lock-in amplifier is being used to check oscillator to high precision in short measuring interval

where ω is the angular frequency associated with $R(t)$. The phase of the reference signal can be chosen so that the coefficient of the $\cos\omega t$ term (A_1) in the expansion is zero, giving

$$R'(t) = A_0 + \sum_{2}^{\infty} A_n \cos n\omega t + \sum_{1}^{\infty} B_n \sin n\omega t \qquad (2)$$

Assume the signal input is

$$S(t) = C \sin(\omega t + \delta) \qquad (3)$$

After passing through an idealized low-pass filter with $(\sin f)/f$ frequency response and a cut off frequency much lower that ω, the output E_o will be

$$E_0 = \frac{1}{T}\int_{\tau}^{T+\tau} R'(u)\sin(\omega u + \delta)\, du \qquad (4)$$

where T is the integrating time of the filter. When intergrated

$$E_0 = \frac{C A_1 \cos\delta}{2} \qquad (5)$$

If δ is 0 (the signal is in phase with the reference) the output is maximum and has positive polarity. If phase angle δ is 180 degrees, the output is again a maximum but of opposite polarity. If δ is 90 degrees there is no d-c output. For any phase angle the output is a function of both the amplitude and phase of the signal and the amplitude of the fundamental component of $R'(t)$. Therefore the circuit is often called a phase sensitive detector.

A phase sensitive detector is sensitive not only to a particular frequency, but also to a particular phase component of this frequency. In lock-in amplifier applications both or either property may be exploited.

The general problem is to recover a weak signal obscured by noise. Noise arises from such statistical fluctuation phenomena as Johnson

noise in resistors and shot noise in vacuum tubes and semiconductors; these both produce a white noise spectrum, in which the noise power per unit bandwidth is the same at all frequencies. Another source is the so-called gain modulation or flicker effect noise associated with both vacuum tube and transistor amplifying circuits. This noise frequency spectrum varies as $1/f$; a large contribution to the total noise occurs near d-c. Interference phenomena, which are not really noise but produce the same effect in obscuring the signal one desires to detect, include power line pick up and r-f interference.

With respect to white noise, little can be gained by moving the signal frequency to a different value. However, the contribution of the white noise to the output voltage of the detection system is inversely proportional to the square root of the bandwidth, and can be reduced to an arbitrary small value by reducing bandwidth. For $1/f$ noise and interference, an operating frequency different from d-c and from interfering frequencies can be selected.

Figure 1 shows a general purpose lock-in amplifier with a tuning range from 15 to 15,000 cps and a variable bandwidth down to 0.12 cps. The front panel of the instrument is shown in the photograph. The input signal, including its associated noise, is passed through a narrow-band tuned amplifier; this initial stage, though of relatively wide bandwidth, reduces noise signals that could overdrive the phase-sensitive detector, thus allowing a larger output before non-linearity is encountered.

Furthermore, the waveform with which the signal is mixed usually contains harmonics of the reference frequency. The tuned amplifier is useful in preventing these harmonics from reaching the detector.

If the signal is only a small percent of the input to the detector, and if its peak value (signal plus noise) is within the linear range of the detector, detector output is small but can be amplified; this is accomplished by the d-c amplifier shown in Fig. 1.

In the reference channel (also Fig. 1) a variable phase shifter controls the phase between reference and signal. The phase sensitive detector used in Fig. 1 is essentially a dpdt switch requiring a square wave drive obtained by applying the reference signal to a Schmitt trigger. The waveform applied to the input of the phase shifter must be nearly sinusoidal and thus a tuned amplifier is also provided in the reference channel; The tuned amplifier can be switched out of the reference channel when a nearly sinusoidal reference signal is available. Also, the reference channel tuned amplifier can be used with a positive feedback loop as an oscillator, simultaneously driving the phase detector and providing a sinusoidal output. Thus, the phase detector can be synchronized to an external frequency or the reference frequency can be generated internally. The amplifiers are gang-tuned over the full operating range, which is 15 cps to 15 Kc in three ranges.

The lock-in amplifier has many uses. In radio astronomy it enables stellar noise signals as much as 40 db below the input noise level of the receiver to be detected and measured. It has been used to compare the frequencies of oscillators within one part in 10^9, in only a few minutes measuring time. It is able to detect the small change in the absorbed losses of an r-f coil used in nuclear magnetic resonance experiments. It can be used as a narrow-band spectrum analyzer to detect and measure a particular Fourier component in a signal spectrum.

The weak noise signal picked up by the antenna of a radio telescope is similar to the noise generated in the radio receiver and is indistinguishable from it. Moreover, the level of the stellar noise is much lower than the effective input noise of a typical microwave receiver. A microwave radiometer, shown in Fig. 2A, designed to apply the lock-in amplifier technique to this problem was described by R. H. Dicke in 1947[1].

The input of the receiver is alternately switched between the signal from the antenna and a laboratory noise source (a warm resistor generating thermal noise). The microwave switching device is driven by a small synchronous motor and the driving frequency applied to the synchronous motor

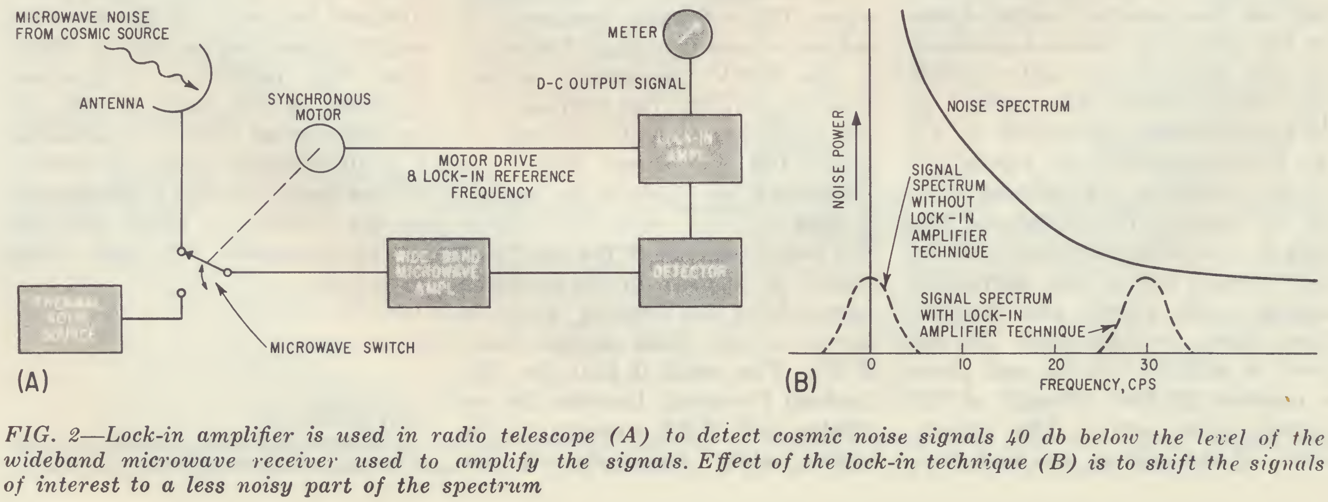


FIG. 2—Lock-in amplifier is used in radio telescope (A) to detect cosmic noise signals 40 db below the level of the wideband microwave receiver used to amplify the signals. Effect of the lock-in technique (B) is to shift the signals of interest to a less noisy part of the spectrum

is also applied to the lock-in amplifier as reference. If the level of noise from the antenna is different from the laboratory noise source, the switching produces a noise signal whose amplitude varies at the frequency used to drive the synchronous motor. This modulated noise signal is then applied to the input of the microwave receiver, which consists of a wideband amplifier followed by a detector. The output of this amplifier consists of two components: one is the modulated noise signal produced by the switch; the other, usually much larger, is the noise generated at the input of the microwave amplifier itself. This combination is applied to the detector.

Consider the output of the detector in a small interval about the frequency at which the microwave switch is driven. All the frequency components of the modulated noise signal carry sidebands corresponding to the frequency at which the switch is driven; thus all these frequency components contribute to the output of the detector. The contribution of the noise produced at the amplifier input to the output of the detector at this frequency is almost entirely due to intermodulation between the various frequency components of this noise signal. The result is, while essentially all the modulated noise signal contributes to the detector output in this frequency interval, only a small fraction of the internally generated receiver noise contributes to it. The output of the detector is then fed into the input of the lock-in amplifier.

The lock-in amplifier selects that part of the detector output lying in the frequency interval provided by the synchronous motor-driven microwave switch and converts it to an equivalent bandwidth at d-c As has been shown, the bandwidth is determined by the low-pass filter at its output. The output of the lock-in amplifier consists of the d-c voltage due to the modulated cosmic noise signal plus fluctuations due to receiver noise. The d-c level is entirely due to, and gives a measure of, the strength of the cosmic noise source. The parameters of the system are such that

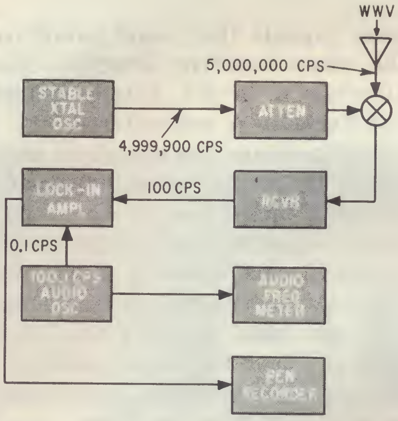


FIG. 3—Oscillators can be checked to one part in 10^{10} in five minutes using lock-in technique

cosmic noise sources that produce a level 40 db below the effective input noise level of the microwave receiver may be detected.

Another application is in the study of a particular phase component of a signal in the presence of a quadrature component. This situation occurs in making a measurement that depends on the balance of an a-c bridge. By setting the phase of the reference signal equal to the phase component of interest, the instrument will reject the quadrature component completely. In the case of balancing an a-c bridge, if one desires to deal with both the in-phase and the quadrature component, the lock-in amplifier can be used to pick up the in-phase and the quadrature components separately and allows each to be balanced separately. This usually allows the bridge to be balanced systematically and avoids the typical successive approximations approach.

An example of an application where the reference frequency is different from the signal frequency is the WWV comparison system shown in Fig. 3. The frequency of the precision crystal oscillator is offset 100 cps from WWV, to 4,999,900.0 cps if the 5-Mc carrier is used.

A small amount of the oscillator output is applied to the antenna terminals of the receiver, which is tuned to the 5-Mc signal from WWV. The result is that the 100-cps beat frequency between the oscillator and WWV appears at the audio output terminals of the receiver; this signal is applied to the input of the lock-in amplifier. The reference frequency for the lock-in amplifier is set to 100.1 cps and the output is thus 0.1 cps, which may be recorded.

In this example the frequency of the oscillator is given by the standard WWV frequency minus the audio frequency, plus the frequency exhibited on the recorder. If a comparison is desired to one part in 10^7 (to 0.5 cps), the audio frequency need only be known to 0.5 percent and the 0.1 cps being recorded may be completely neglected provided it stays reasonably close to this value throughout the entire time of the measurement interval.

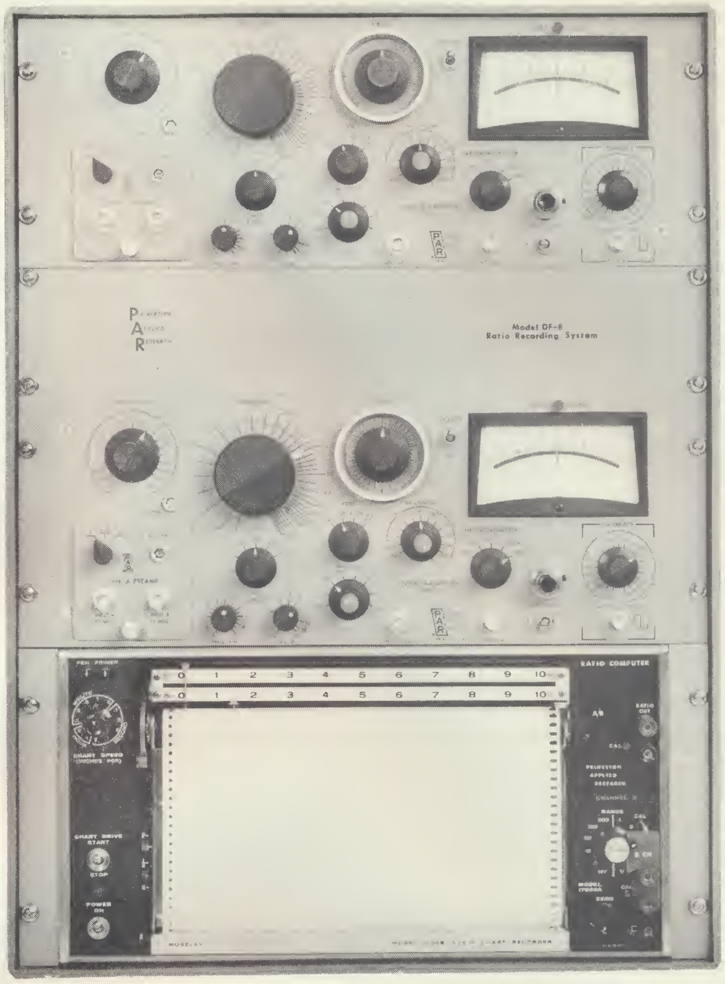
Since this technique involves measurement intervals of about twenty minutes, even inexpensive commercial audio oscillators are adequate for the check. For a high precision measurement, such as a local comparison between two stable crystal oscillators, or between a crystal oscillator and an atomic clock with an accuracy of better than one part in 10, the beat frequency should be recorded for about twenty minutes and a cycle counter and interval timer should be used to measure the time interval for about 10^5 audio frequency oscillations.

Standard commercial interval timers have a precision adequate for this application. Since the recorder allows a phase measurement at the beginning and end of the measurement interval, the accuracy of the measurement can be substantially greater than the total number of cycles in the measurement interval. If, the recorded output phase can be read to better than 0.1 radian, a comparison accurate to one part in 10^{10} can be made in an interval containing 3×10^8 cycles of crystal frequency. At one megacycle this is 300 seconds or five minutes, a short time for a measurement of such high precision.

REFERENCE

(1) R. H. Dicke, Review of Scientific Instruments, 17, p 268, 1947.

MODEL DF-8 RATIO RECORDING SYSTEM



The Model DF-8 Ratio Recording System combines two precision Model HR-8 Lock-In Amplifiers and a high-quality, dual channel recorder into an integrated system capable of recovering two extremely low level signals from noise and monitoring their ratio. Relative measurements of such characteristics as transmission, absorption, reflectivity, fluorescence and luminescence, as well as characteristic response curves can be readily accommodated by the Model DF-8 Ratio Recording System.

Additional flexibility is inherent in this system, since either one or both of the Precision Model HR-8 Lock-In Amplifiers, as well as the recorder, can be used independently in a wide variety of experimental situations. The complete technical specifications and operational capabilities of the Model HR-8 Precision Lock-In Amplifiers are described on the enclosed data sheets.

Ratio Recorder

The dual channel, fast response recorder used in the Model DF-8 Ratio Recording System is a standard laboratory instrument featuring high accuracy and a range of chart speeds. Its operation in the ratio mode has been accomplished by the addition of special circuitry that allows critically damped response over a dynamic range as wide as 100:1. A 0-1 volt DC signal proportional to the measured ratio is also available for digital voltmeter monitoring or other uses. One channel of the recorder can be used without modification as a standard linear strip-chart recorder when the ratio mode is not required.

INPUT VOLTAGE LEVELS:	Determined by the sensitivity settings of the Model HR-8 Lock-In Amplifiers.
RATIO RANGE:	$\frac{A}{B}$: 0.0 to 1.0, B range 0.1 to 10 V.
ACCURACY:	0.25 per cent full scale.
CHART SPEEDS:	12 selectable speeds: 1, 2 in/hr; 0.1, 0.2, 0.5, 1, 2, in/min; 0.1, 0.2, 0.5, 1, 2, in/sec.
RESPONSE:	0.5 sec full scale.
OUTPUT:	0 - 1 V DC output proportional to recorded ratio of 0.0 - 1.0 is available to drive digital voltmeters or other external circuitry.
SIZE:	(System) 26 1/2"H X 18"D X 19 3/4"W Rack Cabinet.
WEIGHT:	Approximately 100 pounds.
PRICE:	$7,500.00. Export price approximately 5 per cent higher (except Canada).

Specifications and Price Subject to Change Without Notice

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